RESOURCE MATERIAL ON CISCE CURRICULUM

MATHEMATICS AT THE PRIMARY LEVEL

Research Development and Consultancy Division

Council for the Indian School Certificate Examinations
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Introductory Note

The Curriculum for Preschool to Class VIII, developed by the Council for the Indian School Certificate Examinations aims to bring about uniformity in the syllabi being transacted in all schools affiliated to the Council and to ensure that the basic minimum standards are maintained. In addition to providing guidelines to schools that are newly affiliated to the Council, the curriculum also endeavours to provide a strong foundation at the elementary level and to facilitate upward mobility so that children derive full benefit of the ICSE and the ISC curriculum.

This Resource Material has been prepared to develop a better understanding on the CISCE Curriculum. It will serve as a supplement to the Curriculum and act as a ready reference material and guide for all stakeholders, including Subject Teachers, Master Trainers, Academic Coordinators and Heads of Schools.

The Resource Material includes a set of documents, which provide a broad understanding on the Curriculum, besides dealing with the teaching-learning strategies related to specific subjects at the Primary and the Upper Primary levels. We have made a beginning by preparing modules of Resource Material for selected subjects included in the Curriculum. It is hoped that over time, we will be able to supplement this Resource Material by developing subject specific modules on more subjects.

I would like to express my special gratitude to Prof. Manju Jain (Former Head, DEE, NCERT), Prof. Sandhya Paranjpe (Senior Consultant), Prof. Anup Rajput (Head, DEE, NCERT), Prof. Kirti Kapoor (DCS, NCERT), Prof. Dharamprakash (DEE, NCERT), Dr. M.S. Dahiya (Senior Lecturer, Retired, SCERT, Delhi) and Dr. Satyavir Singh (Principal, SNI College, Pilana, Baghpat) for their valuable contribution in developing this Resource Material.
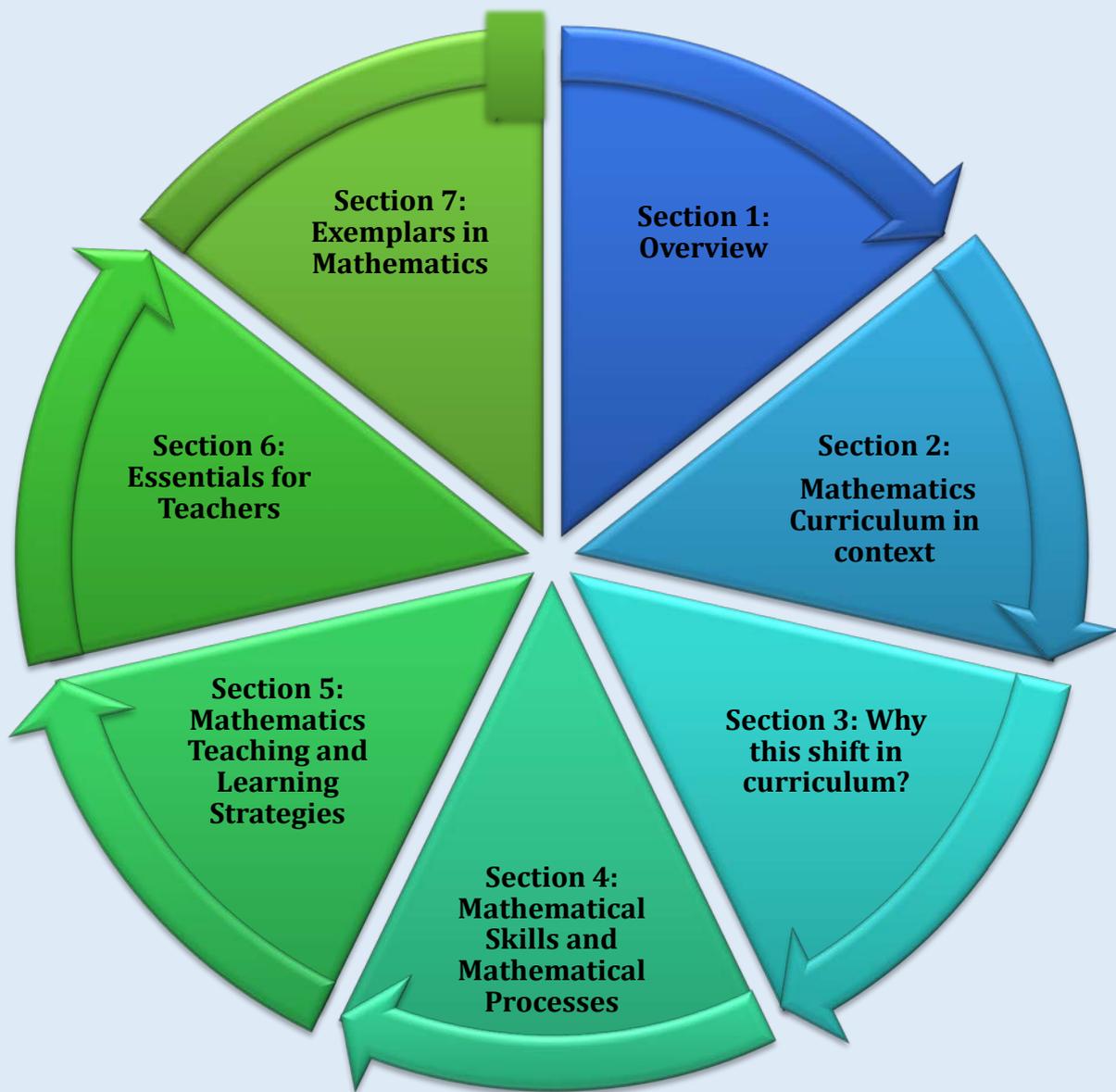
I would also like to acknowledge the teachers, from schools affiliated to the Council, who have been an integral part of this exercise, whose inputs and feedback has helped shape this document.

Last but not the least, I appreciate the efforts put in by Mrs. Shilpi Gupta, Deputy Head (RDCD) in preparing this document along with her team of Dr. Manika Sharma, Dr. M.K. Gandhi, Ms. Mansi Guleria and Mrs. Roshni George.

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Chief Executive & Secretary

MODULE II

Content Outline

Section 7: Exemplars in Mathematics

Section 1: Overview

Section 6: Essentials for Teachers

Section 2: Mathematics Curriculum in context

Section 5: Mathematics Teaching and Learning Strategies

Section 3: Why this shift in curriculum?

Section 4: Mathematical Skills and Mathematical Processes

Structure of the Module

1. An Overview:
   1.1 Why this module?
   1.2 What does this module include?
   1.3 How to use this module?
2. Mathematics curriculum in context
   2.1 Nature of Mathematics
   2.2 Present status of Mathematics teaching-learning
3. Why this shift in curriculum?
4. Mathematical Skills and Mathematical Processes
5. Mathematics Teaching and Learning Strategies
6. Essentials for Teachers
   6.1 Mathematical Pedagogical Content Knowledge
   6.2 Assessment of Mathematics learning
   6.3 General guidelines for Mathematics Teachers
   6.4 Some common misconceptions and errors
7. Exemplars in Mathematics
   7.1 Need for exemplars
   7.2 Planning for the teaching learning process
   7.3 Exemplar I
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Expectations of this Module

After reading/using this module the user/teacher will be able to:

- explain the need and significance of this module;
- discuss the nature, status and explain the salient features of the Mathematics curriculum;
- understand the pedagogical processes involved in Mathematics teaching-learning;
- get an insight into the classroom interaction process through exemplars;
- integrate assessment with the teaching learning process;

1. An Overview

Why, What and How of this module

1.1 Why this module?

This module provides an understanding on various aspects of the Mathematics curriculum such as its status in early classes (Classes I to V), salient features and pedagogical principles for Mathematics learning and how a Mathematics classroom can be transformed into a joyful classroom. It will also provide a broader understanding on various components of the Mathematics curriculum such as learning outcomes, identified concepts and sub concepts in the themes, transactional processes and learning resources to be used in the classroom. In order to provide a comprehensive understanding of mathematical learning, some exemplars have been given.

1.2 What does this module include?

This module has seven sections. The first section provides an overview to explain the why and what of the module along with the process of using it. Section 2 discusses the status and nature of the Mathematics curriculum. Section 3 explains the evolution of the present curriculum. Section 4 in highlights various mathematical skills and processes. Section 5 deals with strategies involved in Mathematics teaching and learning process. Section 6 deals with the essentials that are required for Mathematics teachers. Section 7 focuses on exemplars.

1.3 How to use this module?

This module is meant for all stakeholders working at the primary level in general and for Mathematics practitioners in particular. In each section of this module, some text assignments/activities have been given. While reading/using each section, assignments need to be done parallelly. Later these assignments can either be assessed by your mentor or by peers. Secondly, some exemplars are given. These exemplars may be used during training, either in simulation or in actual classroom situation. After using the exemplars, peer reflection is essential as it would generate further ideas for innovation.

2. Mathematics Curriculum in context

The Mathematics curriculum at the primary level aims to develop a number of mathematical skills and processes in children. The curriculum is designed to ensure that children build a solid foundation in mathematics by connecting and applying mathematical concepts in a variety of ways and situations. To support this process, teachers need to find ways and means of integrating concepts from various themes and apply mathematics to real life situations in children's daily lives.

While transacting this curriculum we need to remember that each child is unique in terms of her/ his likes, dislikes, interests, dispositions, skills and behaviour. Each child learns and responds to learning situations in his or her own special way. The teaching strategies at the primary level must address the needs of all children.

The curriculum and the teaching strategies in the initial stages at the primary level address the complex needs of all children to acquire number sense, develop spatial thinking and understand the relationships between objects and location in a three-dimensional space, realize the need for units to measure, and explore data through first hand observations. Keeping these in focus, the curriculum for these classes has been organized through various topics that have strong interconnections.

Curriculum and Syllabus

The general notion of the people regarding the term 'curriculum' is that it denotes subject-wise list of topics to be taught in a particular class. Actually curriculum means all experiences provided to the children in as well as out of school for attaining the educational aims. In general, everything that happens in the school is part of the curriculum. Some aspects of the curriculum may be educational aims, educational experiences to attain these aims and curricular expectations, organisation of educational experiences and assessment whether these aims are being attained appropriately. Curriculum can be viewed as whole, including the learner, the teacher, teaching and learning methodologies, experiences, outputs and outcomes possible within a learning institution. Curriculum can be considered as content, process and product.

A syllabus is an outline of the content which has to be learnt by the children. It includes the topics to be covered, their order and any other relevant information. Curriculum prescribes the objectives while, the syllabus describes the means to achieve them. Hence we can say curriculum is prescriptive while syllabus is descriptive.

The present Mathematics curriculum expands spirally
from Class I to Class V. For example, let us take the concept of Numbers. The child picks up number skills gradually, with numbers upto 99 in Class I to seven digit numbers in Class V. The understanding of place value develops along with it. Similar approach percolates down to all other themes in the curriculum.

In order to create a classroom where the learners enjoy while learning, where they feel free to ask questions, to explore and create self-learning, the teacher needs to act as a motivator, guide and facilitator and provide children with positive experiences, so that the fear and anxiety that a number of children face regarding Mathematics may be addressed.

2.1 Nature of Mathematics

Mathematics serves as a tool for both logic and creativity. Thus, it is pursued both for practical utility as well as its intrinsic interest.

There are certain aspects of nature of Mathematics which directly/indirectly impact the teaching learning of Mathematics. It is therefore very important for a mathematics teacher to know about the nature of mathematics so that she can use them for improving learning of Mathematics.

Mathematics by nature is abstract. All themes in mathematics, which include numbers, shapes around us, problem solving patterns and systematic reasoning are ideas that ultimately develop in our mind. Understanding of these abstract ideas is demonstrated by applying them in daily life for various purposes including problem solving. The mathematics curriculum treats mathematics both as a tool for practical utility as well as a discipline that develops reasoning and analytical abilities.

Mathematical ideas grow from "concrete to abstract" and "particular to general". For example, when we observe various objects like a ball, an orange, a melon, etc. we realise that they all have a common attribute of "roundness" which is not connected to their specific attributes like colour, material they are made of, etc. Slowly the property of 'roundness' is further abstracted into the ideas of a sphere. Now, we need not think of an object when we are communicating about a sphere. Hence, through the concrete experiences, the concept of a sphere has been abstracted. This has strong implications for teachers. Teachers need to start with providing concrete experiences to the children before they can introduce symbols and abstract ideas.

Mathematical concepts/ideas are hierarchal in structure. This means that each idea is contained in the idea that follows after it. Let us understand it with an example. From counting of concrete objects, the idea of natural numbers is abstracted. By including zero, the set of whole numbers evolves. This gets further enlarged to include negative numbers and we get a set of Integers. Similarly, the set of rational numbers are evolved. This clearly implies that the teacher needs to make sure that the children have understood the previous concepts well before she embarks upon new ones.

Mathematical statements are clear and precise and do not leave any ambiguity. But many a times we use mathematical terms loosely and it brings in impreciseness. For example, use of word "half" in day to day communication. It clearly implies that children need to learn to be clear and precise in their mathematical communications.
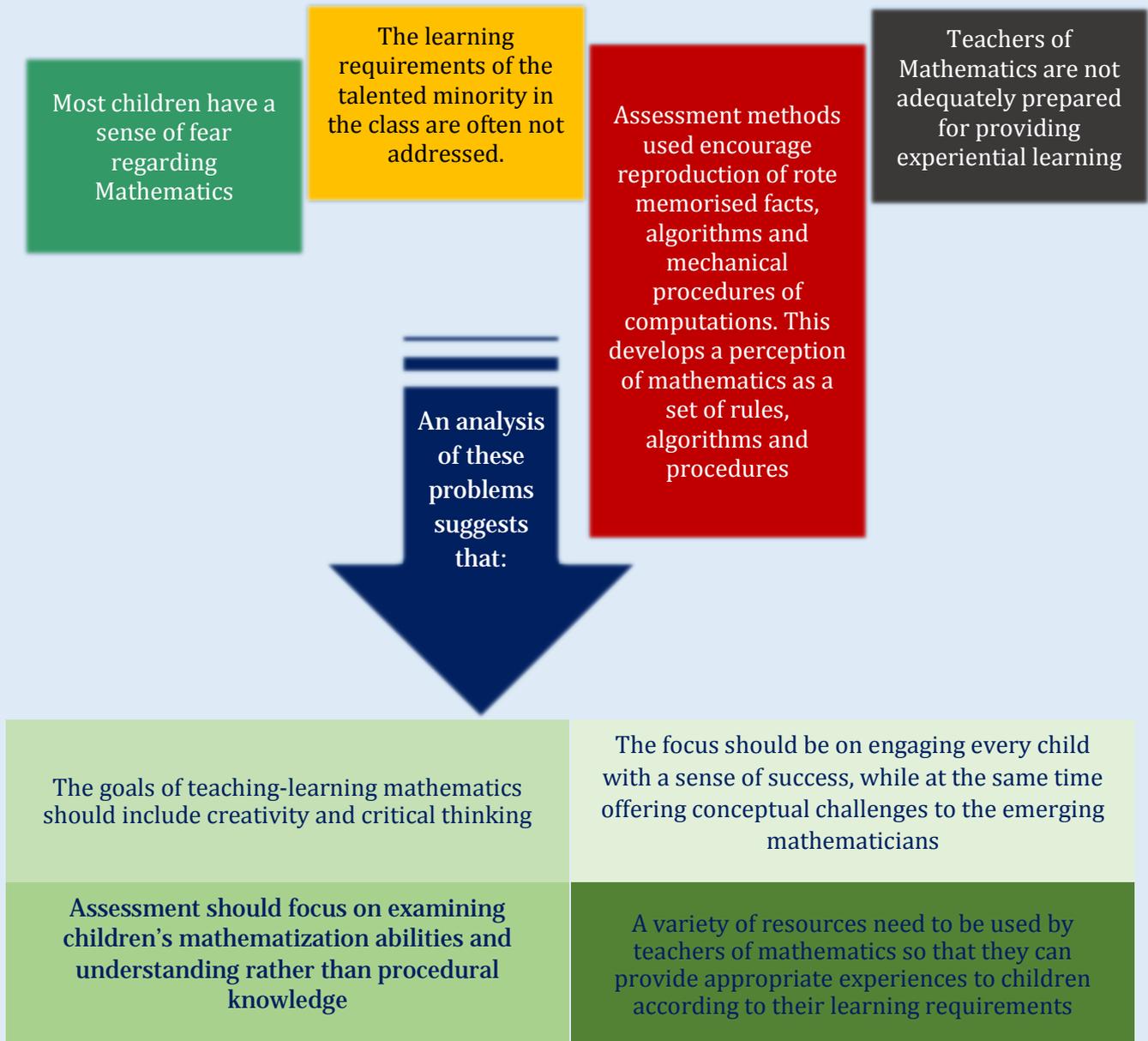
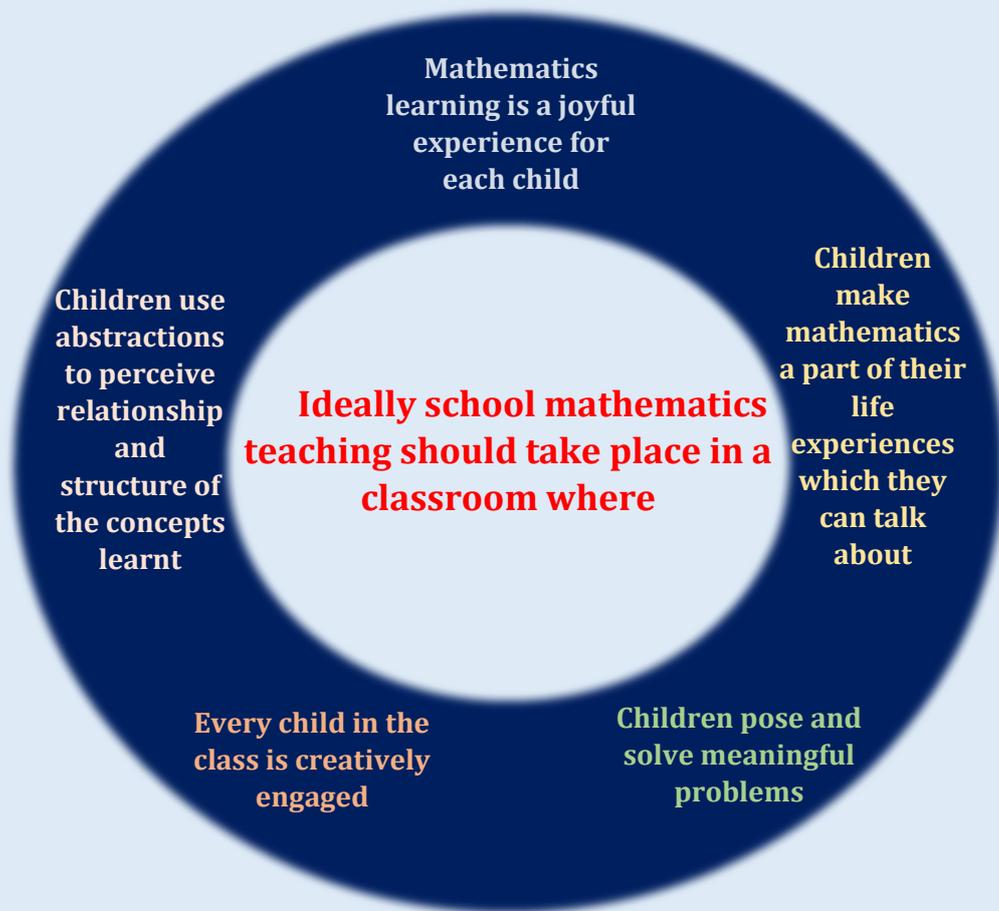## 2.2 Present Status of Mathematics Teaching-Learning: A review

A general tendency in learning mathematics is to memorize mathematical facts and manipulate numbers without having any understanding of the concepts or processes involved. Memorization of rules and mastery of computational algorithms may help children in using Mathematics in their daily life but a conceptual understanding in Mathematics not only helps them apply it in daily life but also develop analytical and critical thinking and better computational skills.

There is a need to help both teachers and children develop conceptual understanding of mathematics. Through this effort mathematics educators can shift from helping children memorize rules to facilitating a deeper understanding of mathematical concepts. Children must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.

Unfolding the Curriculum: Mathematics Curriculum in Practice

Some core areas of concern with respect to the teaching of Mathematics are:

Most children have a sense of fear regarding Mathematics

The learning requirements of the talented minority in the class are often not addressed.

Assessment methods used encourage reproduction of rote memorised facts, algorithms and mechanical procedures of computations. This develops a perception of mathematics as a set of rules, algorithms and procedures

Teachers of Mathematics are not adequately prepared for providing experiential learning

An analysis of these problems suggests that:

The goals of teaching-learning mathematics should include creativity and critical thinking

The focus should be on engaging every child with a sense of success, while at the same time offering conceptual challenges to the emerging mathematicians

Assessment should focus on examining children's mathematization abilities and understanding rather than procedural knowledge

A variety of resources need to be used by teachers of mathematics so that they can provide appropriate experiences to children according to their learning requirements

Mathematics learning is a joyful experience for each child

Children use abstractions to perceive relationship and structure of the concepts learnt

Ideally school mathematics teaching should take place in a classroom where

Children make mathematics a part of their life experiences which they can talk about

Every child in the class is creatively engaged

Children pose and solve meaningful problems

Let us do

Activity-1

1. Why is it essential to understand the nature of Mathematics to achieve the expected learning outcomes? Discuss and elaborate with examples.
2. Write any two objectives of teaching Mathematics in primary classes and substantiate your answer with two concrete examples.

3. Why this shift in curriculum?

The 21st Century is the century of dynamic changes i.e. changes that keep evolving further. A number of skills that were essential in the past decades may not be used to the same extent now. Take for example, the ability to do quick and accurate calculations, which was an essential skill for a student of mathematics – long and complicated calculations are now done by calculators and computing devices at a click of a button! Skills that are in demand in 21st century are creativity, analytical and critical thinking, innovative problem solving and so on. Mathematics as a subject needs to address this essential change by inculcating all such skills in learners and making them ready for the future.
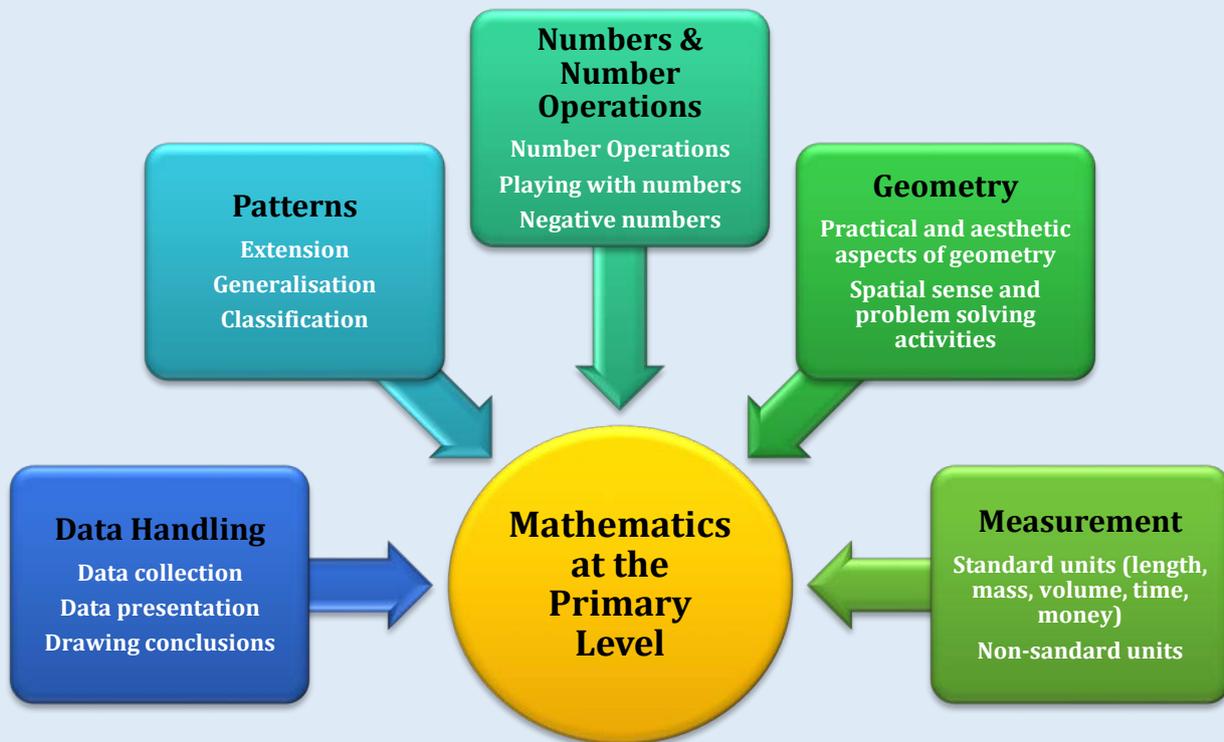
Various skills that need to be inculcated can only be acquired if the curriculum displays pedagogical approaches and encourages experiential learning for the learners. The ability to explore, discover, opportunities to find creative solutions and make mistakes, create new knowledge and rules based on one's own experiences etc., are some aspects of learning which the present curriculum highlights and focuses upon.

Another very important aspect which needs to be kept in mind is the profile of the 21st century learner. Today's learner is curious and ready to ask questions, if given a chance. She gets distracted easily if any activity is continued for long. She is aware of the issues and concerns related to her environment. She is able to use the tools of information and technology and deal with multiplicity in the communication. This change in learning styles and learners' profile is reflected in the present curriculum document which advocates use of multiple strategies for achieving a learning outcome and learning resources to promote experiential learning. The curriculum encourages teaching-learning processes which are child-centric and provides for experiential learning using all possible resources available.

The above change in the learner's profile and the teacher's role (pedagogy) in classroom also suggests a systematic change in manner in which assessment is done. Learners need to be assessed continuously to understand how and what they are learning. Moreover, it is also required that a teacher should continuously assess her classroom processes according to the learning styles of the children and for achievement of desired Learning Outcomes. The teacher needs to keep a record of the child's progress through portfolios and anecdotal records. Focus should be on monitoring learning and helping the child to learn comprehensively.

Implementing such a curriculum will make the classroom a joyful experience, where child is learning by doing and the teacher is continuously assessing the learning and modifying the teaching learning strategies accordingly.

The major themes to be covered, from Classes I to V, are briefly outlined below:

Numbers & Number Operations
Number Operations
Playing with numbers
Negative numbers

Patterns
Extension
Generalisation
Classification

Geometry
Practical and aesthetic aspects of geometry
Spatial sense and problem solving activities

Data Handling
Data collection
Data presentation
Drawing conclusions

Mathematics at the Primary Level

Measurement
Standard units (length, mass, volume, time, money)
Non-sandard units

Learning Outcomes

Learning outcomes are statements that describe significant and essential learning that learners have achieved, and can reliably demonstrate at the end of a unit/lesson or concept. In other words, learning outcomes identify what the learner will know and be able to do by the end of a unit/lesson or concept. Learning outcomes focus on the end result of learning, regardless of how or where that learning occurred.

Thus, these may be considered as the short term goals of Mathematics syllabus for each grade.

Objectives of teaching learning Mathematics at the primary level

The curriculum states the following expectations at the primary level:
- develop mathematical thinking and problem solving skills and apply these skills to formulate and solve problems;
- develop the necessary process skills for the acquisition and application of mathematical concepts and skills;
- recognise and use connections among mathematical ideas, and between mathematics and other disciplines;
- create love and interest towards mathematics;
- make effective use of a variety of mathematical tools (including information and communication technology) in the learning and application of mathematics;
- design imaginative and creative work arising from mathematical ideas;
- reason logically, communicate mathematically, and learn cooperatively and independently;
- argue critically and create newer knowledge.

The above curricular expectations envisage the goals, the child should be able to achieve during the primary schooling. For example, curricular expectations of developing mathematical thinking and problem solving skills may evolve slowly and may manifest themselves in different ways during the primary schooling. The goals set by the curriculum need to be appreciated by the teachers by first understanding them and then trying to achieve them through various teaching learning-strategies. This will also lead to the holistic development of the child as well as develop the child's interest in mathematics. The curricular goals if implemented in the full spirit will lead to the child becoming a 21st century citizen who is capable of logical, analytical thinking and can find a creative solution to problems.
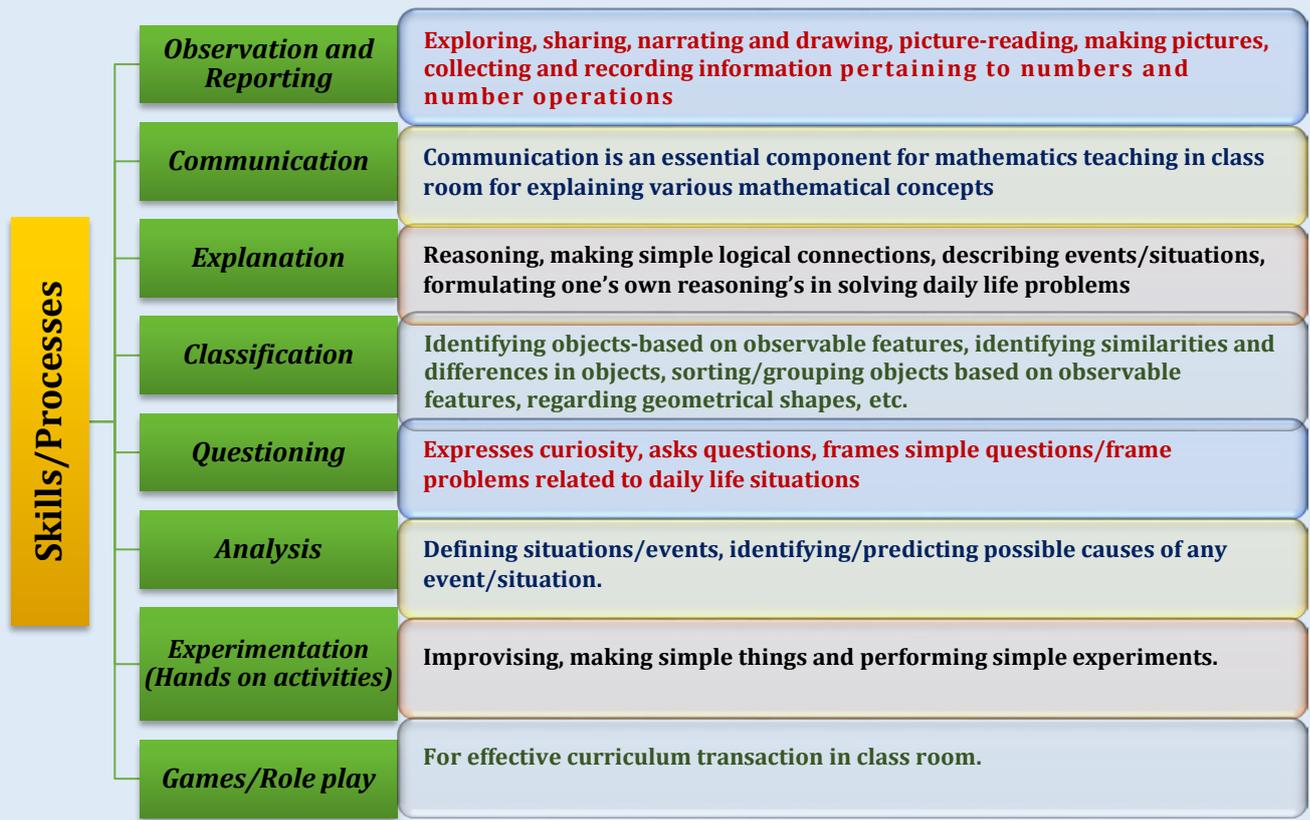
Note: For learning outcomes of Mathematics at primary level, refer to the Curriculum document

4. Mathematical Skills and Mathematical Processes

The present Mathematics curriculum at the primary level aims to develop a number of mathematical skills and processes among children in classes I-V.

Curriculum for classes I to V is designed to ensure that children build a solid foundation in mathematics by connecting and applying mathematical concepts in a variety of ways. To support this process, teachers will, whenever possible, integrate concepts from various themes and apply mathematics to real-life situations in children's daily lives.

4.1 Broad Skills/ Processes that enhance Mathematics Learning

Skills/Processes

Observation and Reporting	Exploring, sharing, narrating and drawing, picture-reading, making pictures, collecting and recording information pertaining to numbers and number operations
Communication	Communication is an essential component for mathematics teaching in class room for explaining various mathematical concepts
Explanation	Reasoning, making simple logical connections, describing events/situations, formulating one's own reasoning's in solving daily life problems
Classification	Identifying objects-based on observable features, identifying similarities and differences in objects, sorting/grouping objects based on observable features, regarding geometrical shapes, etc.
Questioning	Expresses curiosity, asks questions, frames simple questions/frame problems related to daily life situations
Analysis	Defining situations/events, identifying/predicting possible causes of any event/situation.
Experimentation (Hands on activities)	Improvising, making simple things and performing simple experiments.
Games/Role play	For effective curriculum transaction in class room.

Activity-2

Read the Curriculum document and find answers to following questions:

Let us do

- What are changes that have been suggested in pedagogical processes with reference to the present practices?
- Which aspect of the curriculum you find child-centric?
- What changes are envisaged in the role of the teacher?

5. Mathematics Teaching and Learning Strategies

To develop any mathematical concept, teachers need to think about the strategy for classroom interaction. Given below are some suggested teaching-learning strategies with examples, that can be used in classrooms. These are suggestive and should be modified according to children's needs.
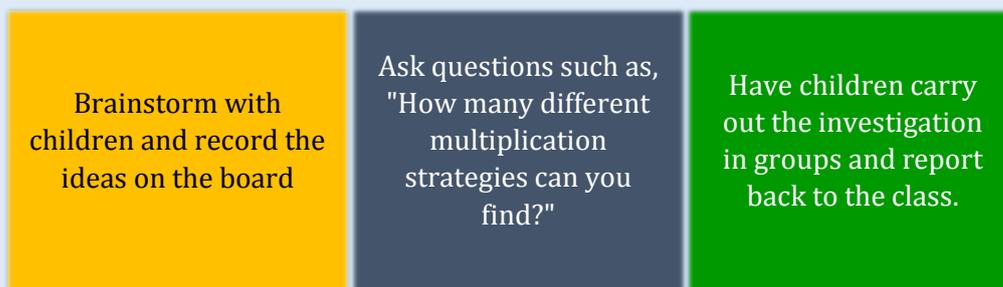
5.1 Experiential learning

Children are natural mathematicians. They push and pull toys, stack blocks, fill and empty cups of water at home. These activities allow children to experience mathematical concepts as they experiment with spatial awareness, measurement, and problem solving. Children learn easily as they describe, explain and consider the ideas from their immediate environment. Experience has an important place in the process of knowledge construction or understanding of a concept. It is an important step in the process of exploration through which individuals can be made to feel, reflect, and arrive at ideas.

For example, when developing knowledge of fractions, a teacher might ask a class to predict where half-way across the table top is. Children would then be invited to make a mark where they think the half-way mark is and write their names next to the mark. When enough children have made their predictions while other children observed, the teacher asks, "How would we work out who is the closest?" Children would then explain how to determine half of the way across the table top and then carry out the measurements to confirm the results. Children could then practise this in pairs, estimating the half-way mark on other objects.

5.2 Problem Based Discussion

Teacher can set a problem or a task for the class to solve. For example, "64 is the answer I have from multiplying certain numbers. How did I get the answer?"

Steps

Brainstorm with children and record the ideas on the board	Ask questions such as, "How many different multiplication strategies can you find?"	Have children carry out the investigation in groups and report back to the class.

To make the learning explicit, it is important that the teacher creates a summary of what has been learnt from solving the problem.

5.3 Open-ended questions

In order to develop better thinking skills in children, we need to ask better questions. What sort of questions do you ask in your classroom? Questions can be either closed or open. Generally, problems can be categorised as under:

Closed questions are used to obtain knowledge or an understanding of facts and have only one correct answer.

For example

'If I have 3 dozen eggs, how many eggs do I have?

This question has 2 closed parts to it. Children need to know the fact that there are 12 in a dozen, and that 3 times 12 is 36.

Open-ended questions involve thoughtful and investigative responses. More than one correct answer is acceptable and children are encouraged to be creative when responding to open-ended questions. Open-ended questions can have variety of possible answers and these allow children to make explorations.

An example of an open-ended question is:
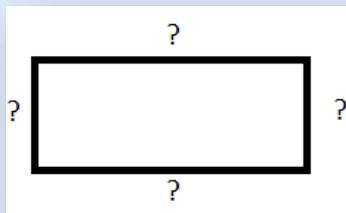
'The total perimeter of the above rectangle is 16 cm. What would be the possible lengths of the sides of the rectangle? How many different rectangles with perimeter of 16cm can you find?'

One answer could be 5 cm and 3 cm. If a child comes up with this answer and stops, ask the class if anyone has a different answer. How many different answers are possible?

Allow the children to discuss their answers in groups and agree upon an answer for presentation and discussion.

5.4 Group work

The purpose of group work is to provide opportunities to children to share ideas and at the same time learn from other group members. Every group should have a leader to observe the group's activities. The leader should delegate tasks to group members and consult the teacher for assistance. Group activities can take place inside or outside the classroom. A good example of a group activity would be drawing shapes such as squares and rectangles, and making models of common three-dimensional shapes such as cubes or cones. Groups of children could also use a cricket or hockey field on which to measure distance and perimeter using traditional methods of measuring such as with strings and sticks.

5.5 Peer Learning

This is organised as a partnership activity in which one child performs a task while the other observes and assists, making corrections and suggesting new ideas and changes. For example, one child decides to multiply a three-digit numbers by a two-digit numbers. The child who is observing should assist and make sure that all the steps are followed before the final answer is given. The teacher's role in this strategy is to observe and encourage positive interaction and effective communication through which the intended outcome can be achieved.

5.6 Projects

Mathematics projects allow children to use a wide range of mathematical concepts in practical/real contexts. Children can apply their understanding, in various activities in which they are participating. For example, children could collect and use traditional materials to make informal measurements, or draw to scale simple maps of the house, school or community.

5.7 Mathematical games

Games are a natural way in which children learn. It is the process through which children explore, investigate, recreate and come to understand their world. Game is an activity in which everything that a child knows and can do is practised or used to make sense of what is new. There are a lot of Mathematics games children can play as part of their learning. For example, a popular game Speak the number aloud can help to clear the concept of division.

5.8 Cooperative learning

Cooperative learning has children working in groups on common problems. The main difference between group work and cooperative learning is that with cooperative learning all children must contribute to the group's learning. A cooperative learning task might ask a group to find all of the different pentominoes i.e. 5 connected squares, which are nets of open cubes. When groups start to work on the problem, challenge them by asking which group has found the most.
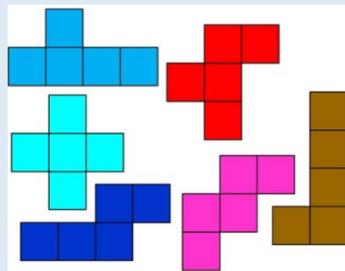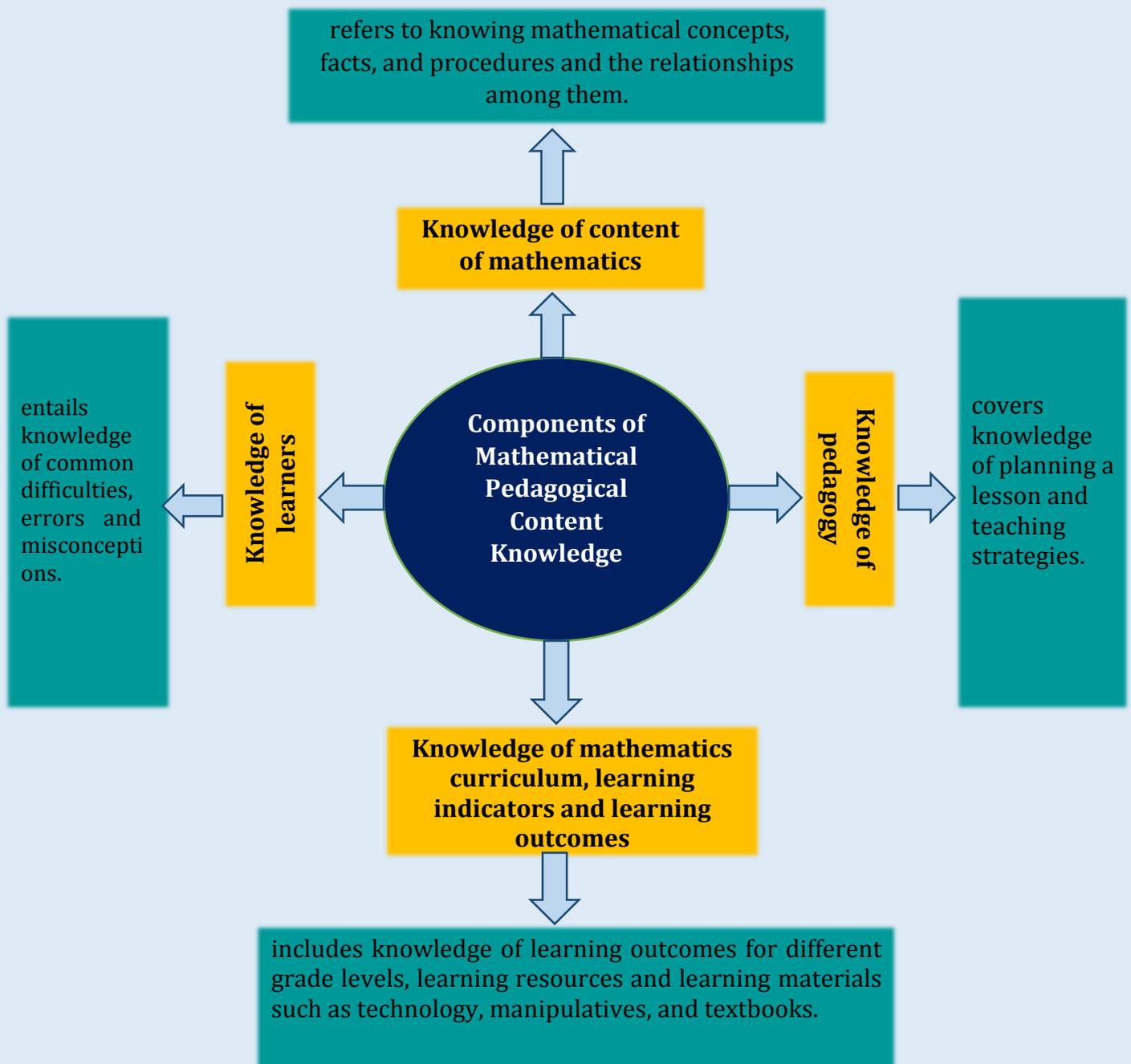

During the teaching-learning process teachers can adopt any one of the above strategies according to the need of concept, prior experiences and social contexts of children. Teachers may use a combination of two or more of the above strategies as the need may be.

6. Essentials for Teachers

6.1 Mathematical Pedagogical Content Knowledge

For a teacher, it is very important that she clearly understands the content that is to be taught. However, knowledge alone of the content will not help without the knowledge of methods using which the content is to be made easily understandable to children. Sound understanding of content and the teaching methodologies is referred to as Pedagogical Content Knowledge (PCK). Teachers need to plan activities according to the need of the concept, level of children and context. Hence, a teacher needs to understand components of mathematical pedagogical content knowledge which are detailed as under:

refers to knowing mathematical concepts, facts, and procedures and the relationships among them.

Knowledge of content of mathematics

Components of Mathematical Pedagogical Content Knowledge

Knowledge of learners

entails knowledge of common difficulties, errors and misconceptions.

Knowledge of pedagogy

covers knowledge of planning a lesson and teaching strategies.

Knowledge of mathematics curriculum, learning indicators and learning outcomes

includes knowledge of learning outcomes for different grade levels, learning resources and learning materials such as technology, manipulatives, and textbooks.

6.2 Assessment of Mathematics learning

The process of assessment involves monitoring children's progress, which enables parents and teachers to adjust instruction to meet children's needs and improve their performance.

During the teaching-learning process, the teacher assesses and monitors the child's learning, focusing on identifying different levels of learning, appropriateness of the activity for the class and/or individual child and finding out what the child has learnt and how. Continuous assessment during teaching-learning provides input/feedback to the teacher to improve her/his teaching strategies.

To address the demands for a broader curriculum and for greater accountability, the teacher needs a variety of tools and strategies. These tools include observations, analytical reviews of children's computational and problem solving work, portfolios, children's self-assessment and traditional and non-traditional paper-pencil procedures.

The teacher may adopt various techniques such as anecdotal records, checklists, rating scales, and scoring rubrics to a keep record of the child's performance. After completion of each unit/theme, teacher may assess the children keeping in view the learning outcomes related to that unit/theme. After an interval (quarter, month etc.), such information can provide a comprehensive picture of the child's learning. The progress made by the children can be communicated to their parents along with the records of their progress. On the basis of this information, the teacher can draw conclusions about performance of both, the individual child and the group as a whole and can adjust his/her teaching strategies.

The major focus of assessment lies on three essential parts; assessment *for* learning, assessment *as* learning and assessment *of* learning. Generally, the first two are termed as formative assessment and the last one as summative assessment. It is important to note that the formative assessment does not mean frequent testing. The above-mentioned tools help in assessing a child's strengths and weaknesses. The gaps in learning found by teachers through continuous assessment need to be addressed through enriching and interesting classroom strategies.

Four stages guide the assessment process; planning the assessment, gathering data and information, interpreting and understanding the data and making decisions based on the data.

'Assessment', 'evaluation' and 'grading' are some terms commonly used. Assessment emphasizes finding out what children know and can do and recording that in a useable form. Evaluation establishes criteria for judging different levels of proficiency or performance, whether the performance is excellent, acceptable, or needs improvement. Grading involves reporting the results of evaluation in some conventional manner, like A, B, C, …. or percentage. The traditional ways of grading say little about what specific skills or concepts the child has demonstrated.

Let us do

Activity-3

How would you assess a group activity in the classroom for learning?

6.3 General guidelines for Mathematics teachers:

All children can be successful with mathematics, provided that they have opportunities to explore mathematical ideas in ways that make personal sense to them and opportunities to develop mathematical concepts and understanding. Children need to know that teachers are interested in their thinking, respect their ideas, are sensitive to their feelings and value their contributions. Teaching of Mathematics needs to focus on children's resources to think and reason, to visualize abstractions and to solve problems. For this, teachers need to take into consideration the following guidelines while teaching Mathematics:

- **Using Real-life examples** – Mathematical problems should be related to real-life situations. Children often ask why mathematics is necessary; relating it to real-life situations will encourage the connection. It is important that teachers provide children with ample time to learn the concept and a sufficient number of opportunities to practice the concept through different contexts which should initiate from familiar contexts and culminate at unfamiliar contexts. Connecting mathematical knowledge to life outside the school and ensuring that mathematics learning moves away from rote methods is important for teachers. Children must be encouraged to relate the mathematics learning to their immediate environment.
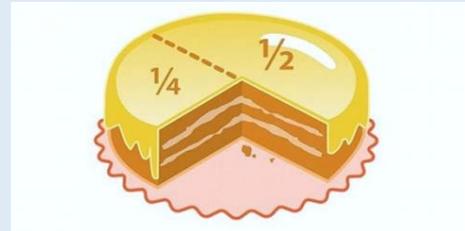
- **Introducing Mathematics as a Language** – Mathematics can be viewed as a language in itself with its own vocabulary and grammar. It must be spoken before being read and read before being written. Some everyday words take on new meanings when used in mathematics and can cause confusion for children, for example, *odd, count, difference*. It is important that the teachers encourage the appropriate and effective use of mathematical language. Children should be able to define and use mathematical terminology which becomes an integral part of their language for communication.

> Add, subtract, divide, multiply, less than, more than, equal to, similar, unequal, fraction, length, breath, height, width, round, oval, square, symmetry, numbers, half, full, parallel, perpendicular

- **Assessing prerequisite skills** – Children must master prerequisite skills prior to learning higher skills.

- **Using concrete materials** – The use of concrete materials or manipulatives will help children in handling the abstractness in mathematics.

- **Promoting a positive attitude toward mathematics** – Teachers must show enthusiasm when teaching mathematics.

- **Using different ways to solve problems**– Although most problems in mathematics are viewed as having only one answer, there may be many ways to get to that answer. Learning math is more than finding the correct answer. It is also a process of solving problems and applying what one has learnt to new problems.
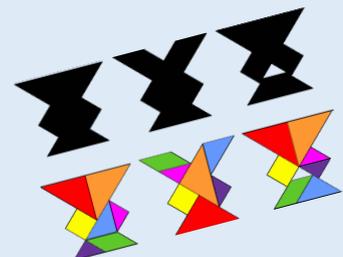
- **Analysing errors and mistakes** – Error analysis is the process of looking closely at child's mistakes to determine what is going on in the child's mind. Error analysis can be done by examining the problems or by talking to children and asking them to demonstrate what they have done. Accuracy is always important in Mathematics. However,

sometimes you can use a wrong answer to help a child figure out why she made a mistake. Analysing wrong answers can help the child to understand the concepts underlying the problem and to learn to apply reasoning skills to arrive at the correct answer.

6.4 Some common misconceptions and errors in Mathematics at primary level

Teachers of mathematics need to be aware of potential misconceptions and errors which may arise when children are learning specific aspects of the subject, be able to recognise them when they occur and also address them successfully.

Children do not come to the classroom as blank slates. Misconceptions frequently arise because children are active participants in the construction of their own mathematical knowledge through reception and interaction of new ideas with the existing ideas.

A misconception is a mistaken idea or view resulting from a misunderstanding of something. **Misconceptions** are, in effect, the misunderstandings about mathematical ideas which children entertain and which usually lead to errors.

> **Teachers need to focus on:**
> - What misconceptions do children have?
> - How do these misconceptions affect mathematics learning?
> - How can these misconceptions be removed?
> - How do we avoid misconceptions?

However, all errors are not the result of misconceptions.

Errors may occur for a variety of underlying reasons, ranging from the careless mistakes (less serious) to errors resulting from misconceptions (more serious).

Common misconceptions in Primary Mathematics

Let us discuss some examples of common misconceptions and their remedies.

Concept: Place Value

Children often recognize multi digit numbers, such as thirty (30) or 400 (four hundred), but they do not understand that the position of a digit determines its value. They may think that the '4' in 46 represents 4, not 40 or 4 tens. Children write numbers incorrectly, i.e.

407 for forty-seven

To remove this misconception, teachers need to focus on place value concept through play money. Children need experiences representing two and three-digit numbers with manipulatives that group (base ten blocks).

Concept: Shapes

Some children may think that a shape is changed by its orientation.

When a square is rotated, its sides form 45-degree angles with the vertical diagonal, hence it is no longer a longer a square but a diamond.

They may see a rectangle with the longer side as the base, but claim that the same rectangle with the shorter side as the base is a different shape.

To remove such type of misconceptions we need to provide experiences with shapes in different orientations. For example, in building-shapes, ask children to orient the smaller shapes in different ways.

It is important to give opportunities to children to handle shapes and physically feel that the shape does not change, regardless of the orientation, as illustrated below.

Teachers need to show children various types of shapes and not just the regular shapes that they see in pattern blocks and on posters.

Concept: Measurement

Children often focus on the size to determine estimates of weight. They can be confused by a big fluffy object and a tiny dense object.

Because children cannot tell actual mass until they have handled an object, it is important that teachers do not ask children to estimate the mass of objects until they have had the opportunity to lift the objects and then make an estimate of the mass.

Concept: Using and reading a ruler

Some children may view the measurement process as a procedural counting task. They might count the markings on a ruler rather than the spaces between (the unit of measure).

Have children use informal or standard length units to make their own rulers by marking each whole unit with a number in the middle.

They will see that the ruler is a representation of a row of units and thus focus on the spaces.

Some children might think that they can only measure lengths with a ruler starting from the left edge. Give them numerous experiences of measuring lengths with child-made tapes or rulers with numbers in the centre of the spaces.

Provide situations where the ruler does not start at zero. For example, a ruler is broken and the first inch number that can be seen is '2'. If a pencil is measured and it is 9 inches on this ruler, the children must subtract 2 inches from the 9 inches to adjust for where the measurement started. Some children may be confused when the ruler they are using has both customary (inches) and metric measures on it. By covering the inches on scale with masking tape the child will be less confused.

Concept: Time

Some children might confuse the hour and the minutes' hands. For example, the time of '3:45 may be read as 9:15. Also, some children name the numeral closest to the hands, regardless of whether this is appropriate. For instance, for the time of 3:45 they may write the time as 3:09 or 9:03.

One way to avoid this confusion is to use strategy of using a one-handed clock to begin telling time. This gets children to focus on the hour's hand first. It also helps them understand that the hour hand gives the most significant information when telling time.

Assess children' understanding of the role of the minutes and hours' hands and the relationship between them.

Provide opportunities for children to experience and measure time to the nearest five minutes and the nearest hour. Have them focus on the movement and features of the hands on real or geared manipulative clocks.
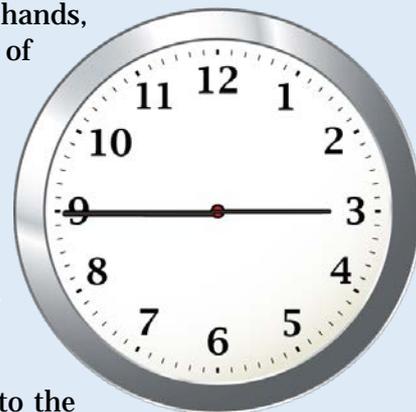
Concept: Addition and Subtraction of Decimals

Children might compute the sum or difference of decimals by lining up the right-hand digits as they would for whole numbers.

For example, in computing the sum of 25.43 + 21.7, children write the problem in this manner:

$$25.43$$
$$+\ 21.7$$
$$27.60$$

To help children add and subtract decimals correctly, have them first estimate the sum or difference. Providing children with a decimal-place value chart will enable them to place the digits in the proper place.

Concept: Division and Place Value

When dividing 1002 by 2, some children get the answer as '51', as they do not consider the place value properly.

To remove this type of misconception teachers need to use the place value concept while dividing any number. Children need to understand that every place must be divided. If there are no such units to make groups as required, they need to write '0'.

Like above there are many more misconceptions in mathematics which result in children committing errors while solving problems or applying mathematical concepts in daily life. We can prevent or minimize many common misconceptions and effectively address those that still emerge, provided our teaching-learning process consistently probes children's understanding and provides opportunities for them to show and explain their reasoning.

7. Exemplars in Mathematics

7.1 Need for exemplar

- The underlying idea for exemplars is to enable practitioners to translate the curriculum into practice in the classroom.
- It will help teachers to understand the different components of the teaching- learning processes in a sequential manner.
- It would also help in developing an understanding of how to achieve the Learning Outcomes

7.2 Planning for the Teaching- Learning Process

- Selecting a theme
- Brainstorming for ideas
- Identifying learning outcomes and sub competencies to be developed for selected concepts
- Selecting learning resource materials
- Selecting activities & pedagogical processes to develop concepts/skills.

7.3 Exemplar I   Class III

Theme:	**Number Operations**
Sub Themes:	Multiplication of single digit numbers forming tables of multiplication facts.
Learning Outcomes:	Children will be able to: 📝 construct and write multiplication tables up to 10.
Key Concept:	Construction of multiplication tables.
Sub competencies:	• Appreciating that multiplication of whole numbers is a shorter way of writing the situation of adding a number many times repeatedly, by using the symbol 'x' for 'times'. • Finding the result of multiplying two numbers by repeated addition, facts like 2x 5 is 2 times 5 which is 5+5=10. • Finding the result of multiplying two numbers by using earlier learnt multiplication facts like 7×6 can be obtained by adding one or more 6 to 6×6. • Construction of multiplication tables by repeated addition, skip counting, observing, and generalizing patterns, using concrete objects for repeated addition like straws. • Differentiating between the meaning of a $\times$ b and b $\times$ a and also the similarity of the net result like 3×4 is 3 times four i.e., $4 + 4 + 4$ and 4×3 is 4 times three i.e. 3+3+3+3.
Suggested transactional processes:	Encouraging and facilitating children to develop multiplication tables (by repeated addition) rather than by rote learning. *** These have been described in detail below.**

For the teacher:

Activities shown below give an idea of some pedagogical processes that can be used to achieve the learning outcomes as mentioned above. While these activities have been given in a sequence, innovative teachers may evolve their own sequence and add on more activities which are suitable to the needs of their children.

Suggested Transactional Processes:

To achieve the learning outcomes following pedagogical process is suggested. These activities may be utilized in any sequence depending on the classroom context, level of the children, etc.

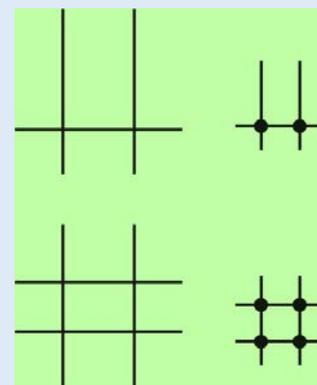**Activity 1: Constructing Multiplication Tables using straws**

Objective:
To construct multiplication tables of 2, 3, 4 and 5 using straws.

Learning resource:
Different sizes of straws, sticks, etc.

Steps

- The teacher with the help of the children may show the initial steps and then let the children take over the development of the multiplication table.
- Let us put two long straws near each other for developing multiplication table for 2.
- Now put a small straw across these two straws.
- Count the points of intersection.
- There are 2 intersections which shows 1 times 2 = 2 or $1 \times 2 = 2$ by writing it on the board.
- Similarly ask the children to put two small straws and count the number of intersections. Then write 2 times 2 or $2 \times 2 = 4$.

Let a group of children now take over and develop the table of 2 using more small straws. Let one of the children take up the responsibility of writing the results on the board with another one checking the results being written. A group of 4 children can easily share responsibilities of developing multiplication tables (as a group assignment).

The teacher may encourage the children to go beyond 10 times of the number (as the standard practice is stopping the multiplication table at 10). It would be rather nice if the children develop skills and ability to construct tables up to say 20 times as it will make them more proficient in multiplications and division later. Also, developing multiplication tables up to any number beyond 10 can give the children a push to words developing number sense also.

Teaching-learning strategies

Individual work/pair work/ group work, observation, experimentation

Discussion in general about multiplication tables
- Learning multiplication tables through rote memory is not advisable at all.
- Children need to understand the process of constructing a multiplication table.
- Different contexts and different processes of constructing multiplication tables will help the children remember multiplication facts.
- Encouraging children to go beyond the standard practice (of stopping the table at 10 times) will help them to calculate multiplication with larger numbers (using commutative property).

Activity 2: Constructing Multiplication Table through skip counting

Objective:
To construct multiplication table using skip counting.

Learning resource:
Paper, pencil, ruler

Steps

- Children may be encouraged to create a 10×10 number grid in which numbers from 1 to 100 are written in continuation as shown below.

Start

1	2	3	4	5	6	7	8	9	10
20	19	18	17	16	15	14	13	12	11
21	22	23	24	25	26	27	28	29	30

Start

1	20	21							
2	19								
3	18								
4	17								
5	16								
6	15								
7	14								
8	13								
9	12								
10	11								

- Mark a starting point before the number '1'.
- Now as children to think of a frog (or any other animal that skips one place and jumps to the next, i.e. skips 1 and jumps to 2, then skips one place (3) and jumps to 4.
- Let the children, individually or in pairs and construct the table of 2.
- Similarly, by skipping 2 places, 3 places or 4 places, the children can themselves make/construct tables of 3, 4 and 5.

Teaching-learning strategies

Individual work/pair work/group work, observation, experimentation, deduction

Some children may find it a bit difficult. For such children, instead of a number grid, you may use a number line for constructing multiplication tables.
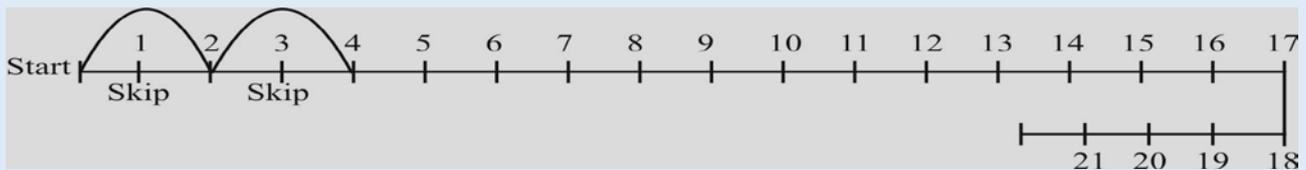
By skipping one place, multiplication table of 2 would be generated. Similarly skipping 2 places, 3 places etc. different tables may be generated.

Discussion points
- Can you see any pattern while constructing the multiplication table?
- If we construct a table of 5, how many numbers will we skip? How did one conclude that?
- To construct multiplication table of bigger numbers, say 8, 9 etc. can skip counting be used? What strategy will you adopt to form such tables?

Activity 3: Constructing multiplication tables using patterns

Objective
To develop multiplication tables based on number patterns

Learning resources:

Paper and pencil

Steps
- Let each child choose one number on which they would like to construct the multiplication table using number pattern.
- Every child writes the numbers starting from 1 in a number pattern depending on the number he/she is constructing the multiplication table on. For example, if a table of 2 has to be constructed then the numbers have to be written in a pattern of 2.

1	2
3	4
5	6
7	8
9	10
....	....
so	on

Teaching-learning strategies

Individual/ pair/ group work, discussion, peer assessment Experimentation, observation

See on the right hand column multiplication table is getting constructed.

Similarly, for a multiplication table of 3 the child would write

1	2	3
4	5	6
7	8	9
10	11	12
so on		:
		:

→ Table of 3 is getting constructed

and for a table of 4 one would write

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	....
17	18	19	....

→ One would see the multiplication table of 4 evolving easily.

- Children can be encouraged to construct tables beyond 10's and 20's using number patterns.
- Once the table has been developed, the children may exchange the tables among themselves and check each other's table for accuracy.

Extension
- Children may be challenged to develop any multiplication table from numbers 6 to 9
- Children may evolve their own patterns to construct multiplication table of 10.

Discussion Points

The teacher may like to discuss the following points

- For writing/constructing multiplication table of 5, how does the number pattern have to be written?
- Why writing number patterns like this constructs a table?

Activity 4: Constructing multiplication tables through repeated addition

Objective
To construct multiplication tables using the process of repeated addition.

Learning Resources:

Materials that can easily be counted, e.g. kidney beans, plastic blocks, tamarind seeds, etc.

Steps
- Provide children with some concrete materials which can be easily counted. For example- kidney beans, plastic blocks, tamarind seeds, etc.
- Let the children interpret
  1 times 2 = ?
  $1 \times 2 = ?$
- Let them put two objects and observe how many are these? Then write
  $1 \times 2 = 2$
- Encourage them to add more objects in twos and write the result
  Two times 2
  or, $2 \times 2 = 2 + 2 = 4$
- Let them keep on doing it further
  Three times 2
  or, $3 \times 2 = 2+2+2 = 6$

Teaching-learning strategies

Group Work, discussion, presentation, sharing with peers, experimentation, observation, deduction

- Let the children develop the multiplication table by repeated addition.
- Similarly, let another group of children construct a table for 3 and present the process to the whole class. The class may raise questions about how they did it, etc.
- With better understanding, children must perform the mental addition like 4×9 i.e. 4 times 9 is 4 less than 4 times 10 i.e. $40 - 4 = 36$
- By the end, through the whole gamut of activities listed above, the children will develop an understanding about multiplication and multiplication tables.

7.4 Exemplar II   Class IV

Theme:	Geometry
Key Concept:	Circle
Learning Outcomes:	Children will be able to: ☑ identify the centre, radius and diameter of a circle; ☑ draw a circle, free hand, using a round object or a compass and identify centre, radius, diameter.
Sub competencies:	The competency of identifying various parts of a circle is developed with along with many other sub competencies like: • Differentiating a circular shape from other shapes on the basis of its characteristics, like, it has no corners and it is a closed shape. • Identifying circular objects from the environment by matching the above characteristics for example: a bangle, wheels, steering of a wheel, face of some clocks/ watches, etc. • Identifying the centre and radius of circle from objects like wheel of a bicycle having spokes attached to the rim. The centre is the point at which all spokes are attached and radius is the length of each spoke. • Drawing a free hand circle, and circle by using the objects identified above, on a plane surface i.e. paper, board, well, ground, etc. and using a thread/ rope on the basis of their observation. • Drawing a circle by using a pair of compass for a given radius. • Observing and generalizing a relationship between radius and diameter of a circle. • Finding centre, radius and diameter of a circular shape cut out drawn on a paper by folding it in different ways to form two equal parts of the shape. Many such activities must be conducted within classroom so that children acquire above sub competencies and finally the competencies that are aimed at in the Learning Outcomes. The activities outside classroom (home assignments) will also help children in observing, experimenting, hypothesizing and finally drawing inferences about circles and its various parts.
Suggested transactional processes:	Conducting paper folding activities which will go a long way in creating a deeper understanding of a circle and the vocabulary related to it. *** These have been described in detail below.**

For the teacher:

To achieve these learning outcomes following pedagogical process is suggested. These activities may be utilized in any sequence depending on the classroom context, level of the student etc.

Suggested Pedagogical Process

Activity 1: Drawing Circles using circular objects

Objectives
- to make different size circles by tracing
- to compare different circles
- to observe the characteristics of a circle

Learning resources:

Circular objects like bangles, bottle caps, glasses, vessels with round edge, etc.

Steps

This is an activity to be conducted by each child.

The children would be given different circular objects and asked to trace out the boundary of edge as a circle on paper using a pencil around the object.

Teaching-learning strategies

Observation, Experimentation, drawing inferences

Discussion points
- The smallest circle is traced by which object?
- The children could be shown the traced circles and asked to match the traced circle with the objects.
- The children could be asked to observe the shape 'circle' and discuss how is it different from other shapes, such as, 'let us see how many corners does a circle have?' The discussion should lead to the characteristics of a circle, that it has no corners, it is closed etc.

Activity 2: Creating the designs using circles by tracing

Objective:
Children will be able to develop
- an insight about the characteristics of a circle.
- visualization and aesthetic sense

Steps
- The children may be asked to draw different designs in their notebook by tracing different circular objects or the same circular object.
- The children should be asked to display their designs and this should be followed by discussion.

Learning resources:

Circular objects like bangles, bottle caps, glasses, vessels with round edge, etc.

Discussion Points
- Which design looks aesthetically better?
- What is special about that particular design?
- What is it that makes a design look attractive?

Activity 3: Study the circles with cut-outs

Objective:
To help the children identify the centre, diameter and radius of a circle.

Learning resources:

Circular objects, paper, scissors.

Steps and Discussion Points
The children could be asked to trace a circle using any circular object and cut it out using a pair of scissors. For younger children or children with special needs circular paper cut-outs may be provided.
- The children could be asked to observe the cut-out circle and to make a crease on the circle by folding it.
- The teacher may ask the children to observe and find out the longest possible crease on the circle. The longest possible crease is obtained when the fold allows the two parts to fully overlap.

Teaching-learning strategies

Observation, experimentaion, deduction

- This longest crease on the circle is the 'diameter' of the circle.
- The child may be asked to draw other creases on the circle by completely overlapping the two parts (i.e. folding along the diameters). The teacher may ask question such as:
  – Are all diameters (longest crease) made on the circle equal?
  – Do all the creases (along diameter) meet at a common point?
  – This point on which the diameters intersect may be named the 'centre' of the circle.
  – Do all circles have a centre?
- The children would be then encouraged to observe that distance from the centre to any point on the boundary of the circle is same. The children could observe this by folding the circular paper cut out and see the folds overlapping.
- This distance from the centre to any other point on the boundary of the circle is the 'radius' of the circle.
- Ask children to find out the relationship between diameter and radius of a circle. With paper folding many children will be able to conclude that diameter is double of the radius or radius is the half of the diameter.
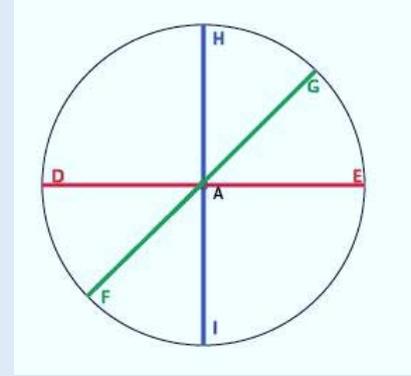
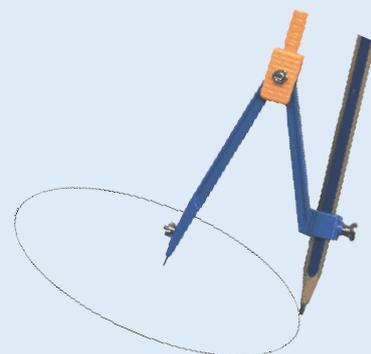## Activity 4: Drawing circles with a pair of compasses

Objectives:
To children would be able to draw circles of various radii using a pair of compasses

Learning resources:

A pair of compasses, paper, scissors.

Steps and Discussion points

- The children would be given a pair of compasses. (The activity could be planned in advance so that children could bring the compasses from home.)
- The discussion similar to the one given below could be done with the children.
- We all observed that circles have a point called as 'centre'
- How is this point related to the circle?
- This point called as 'centre' because it is equidistant from the circle.

- ➤ So, can the tip of the compass be fixed at a point and then without moving the tip, if the pencil is moved around it, what will be get? We will then get a circle.
- ➤ What happens if the distance between the tip of the compasses and the pencil varies or keeps on changing when you are drawing a circle?
- • The children should be given opportunities to draw circles of different sizes by varying the distance between the tip of the compasses the pencil.
- • *Many children may have experience of drawing circular shape during play by using stretched thread/rope. Utilize this experience and ask children to draw circles on the board or floor using the similar technique.*

Activity 5: Creating designs with circles using a pair of compasses
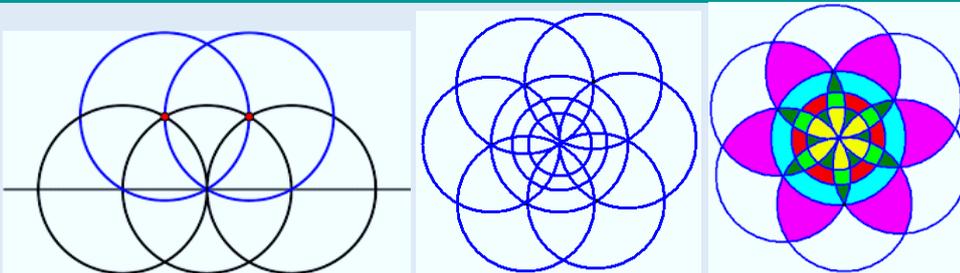
Objective:
Providing experiences for
- • exploring their own designs with a compass
- • practice in drawing circles with a compass

Learning resources:

A pair of compasses, paper, pencil.

Steps and Discussion points

- • The children would be asked to draw designs using compasses by varying the 'centre' point and the distance between the tip of the compasses and the pencil i.e. by varying the radius.
- • The children should be encouraged to share their techniques of making the designs.
- • The discussions should be done on which designs are aesthetically better and how better designs can be formed.
- • The 6 petal design and other similar designs should also be discussed in the classroom.

Additional Activity/ Assignment for applying the skills learnt

Drawing a circle using rope and sticks in the ground

Objective
- The students would be able to identify the centre, radius of the circle
- The student would be able to construct circles with a given radius

Learning Resource:
Wooden sticks, thread/rope.
(The students should be taken out in the playground for the activity.)

Steps:
- Take a piece of rope.
- Tie a stick on one of its ends
- Press the other end of the rope on ground by your thumb or foot.
- Move the stick on the ground keeping the rope stretched. The mark put on by the stick will form a circle

Teaching-learning strategies
Experimentation, observation

Discussion Points
- Why did we get a circle on keeping the rope stretched? If the rope is not stretched what shape can we obtain?
- Mark the centre and radius of the circle.
- Can you find the diameter of this circle? If yes, how?

Let us do

Activity-4 (Group Work)

What are the three takeaways of this module for teachers, administrators and master trainers.

Make a list.